**Exercise 1:**

1. To prove that is a constant of motion, we have to take the derivatative w.r.t time and prove it equal to zero.

We know that , then

Also,.

Since, is left eigen vector of . Therefore,

This proves that is a constant of motion.

1. Graphs (a), (b) and (d) will converge because all the three graphs contain rooted out branching. Yes, the **consensus value** depends on the initial condition. **However, the convergence(whether the graph will converge or not) doesn’t depend on the initial condition.**
2. For graph (a):

Initial condition = [20, 10, 15, 12, 30, 12, 15, 16]

Agreed Value = [15.5]

For graph (b):

Initial condition = [20, 10, 15, 12, 30, 12, 15, 16]

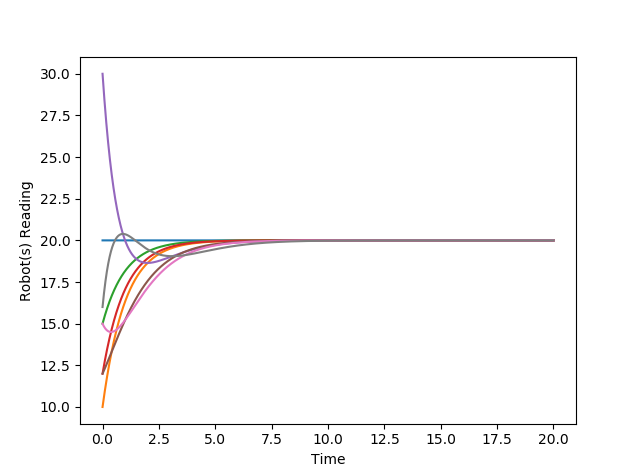
Agreed Value = [20]

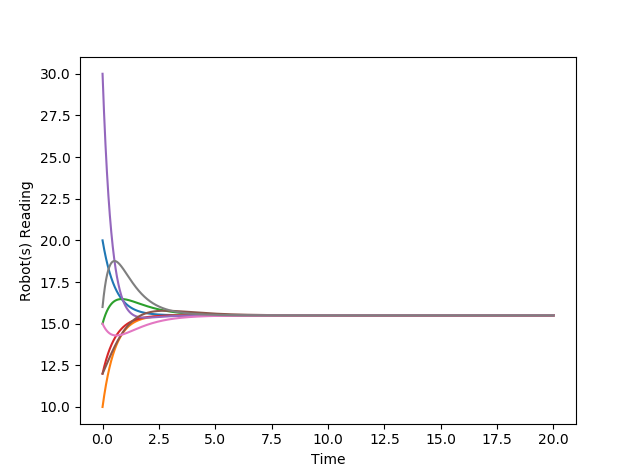
For graph (d):

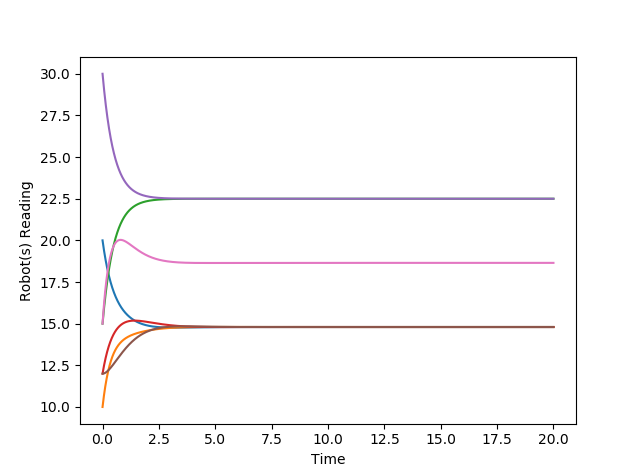
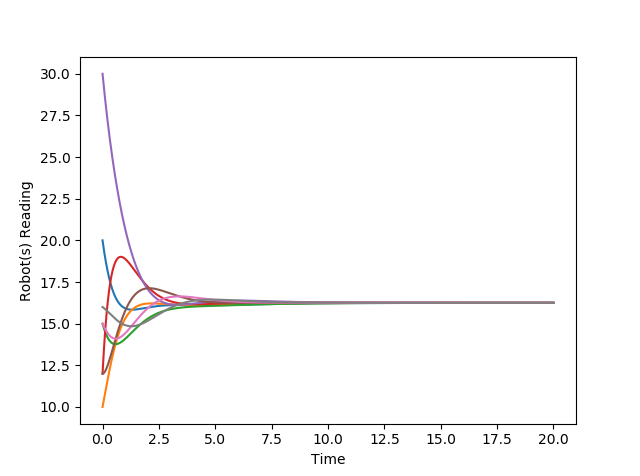
Initial condition = [20, 10, 15, 12, 30, 12, 15, 16]

Agreed Value = [16.25]

1. For the code please refer to the *“hw2\_1\_3.py”* file

Following the plots:

Graph (a) Graph (b)



Graph (c) Graph (d)

**Exercise 2:**

1. To calculate the fixed points of this system, we need to find the solution for

The only solution for the above two equations is

1. Given:

For

Let us evaluate

**Since, and , is positive definite. Therefore, it is a valid Lypunov function.**

For

**Since, i.e. it is not strictly positive definite . Therefore, it is not a valid Lypunov function.**

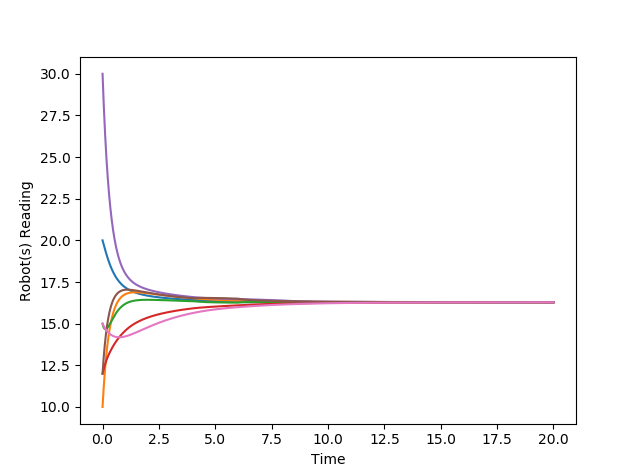
For

Let us evaluate

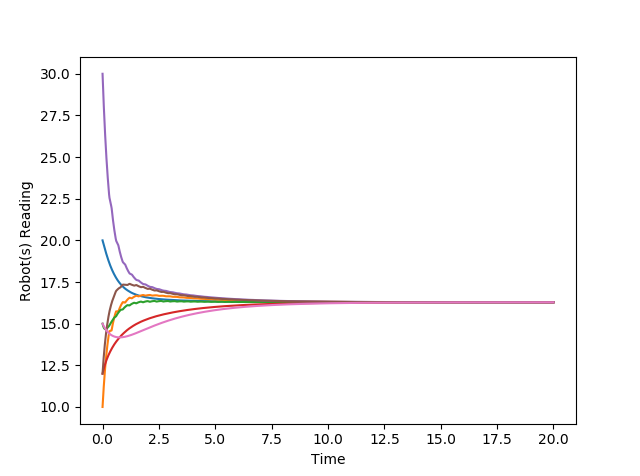
**Since, and , is positive definite. Therefore, it is a valid Lypunov function.**

With  we can say that fixed point is **asymptotically stable.**

**Exercise 4:**

1. **Yes**, for each graph the consensus protocol will **converge** since, in all the cases the graph has **rooted out branching**. Since, it is also a **balanced** **digraph**, the consensus protocol will converge to **average** **value**.
2. **Yes**, even with switching the consensus protocol will **converge**  because at all the time the **union** of all the **graphs contain rooted out branching**.
3. Please refer to *‘simulateConsensusSwitch.py’*
4. ****We can see that with switch time = 2s plot is smooth in comparison to switch time = 0.1s

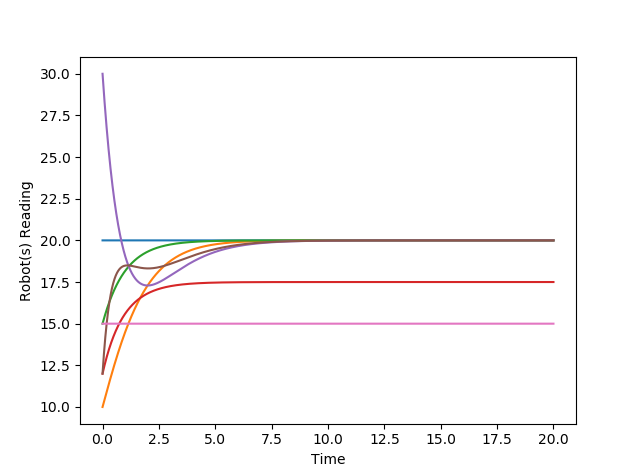
When switch time = 2s



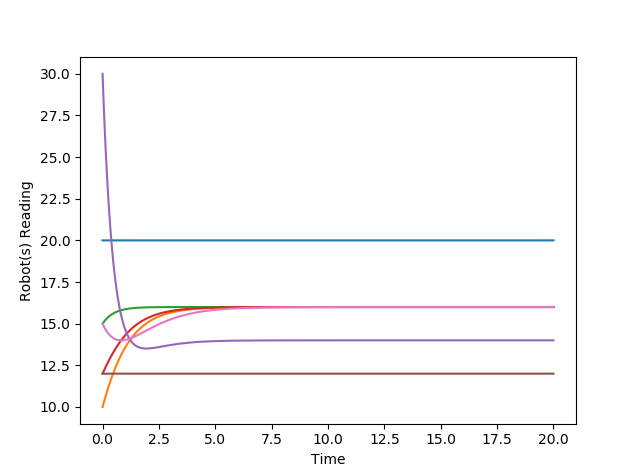
When switch time = 0.1s

**Exercise 5:**

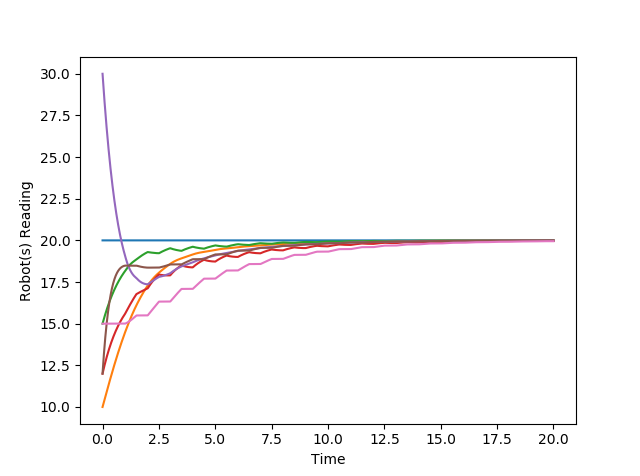
1. **No**, for each graph the consensus protocol will **not** **converge** since, in all the cases the graph **doesn’t have rooted out branching**
2. **Yes**, with switching, the consensus protocol will **converge**  because at all the time the **union** of all the **graphs contain rooted out branching**.

****

1. When no switching,

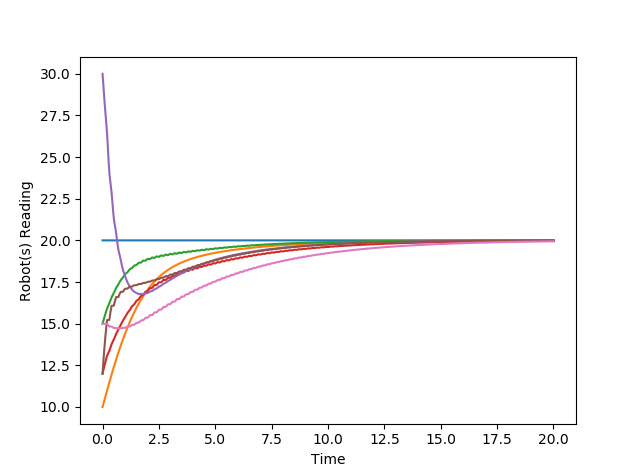
 Graph (a)

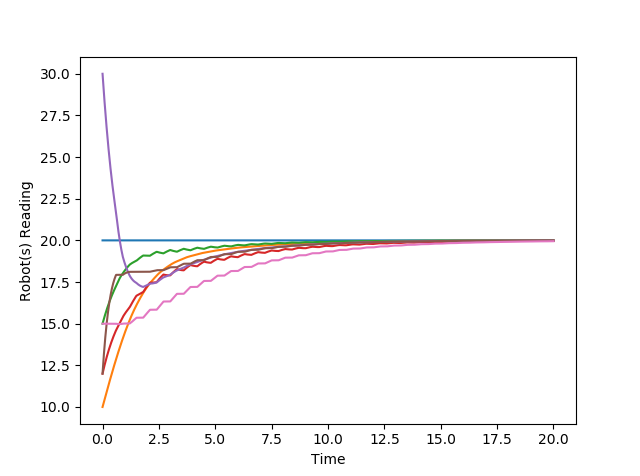
Graph (b)

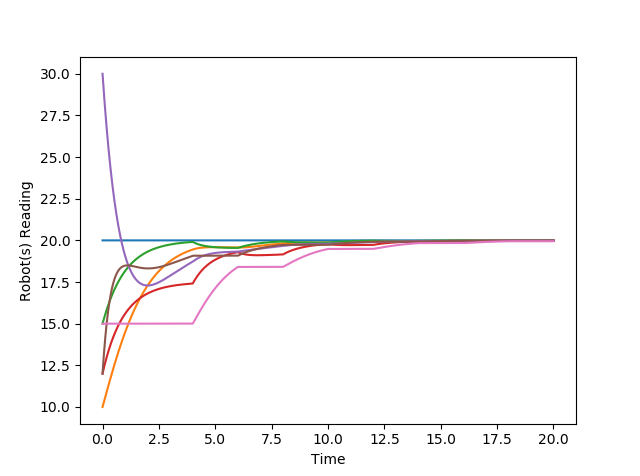
 When switch time = 0.5 s

We ca see that in switching case it is converging while in non-switching case the consensus protocol will fail.

1. Following are the plots

****Switch time = 0.1s

Switch time = 0.3s



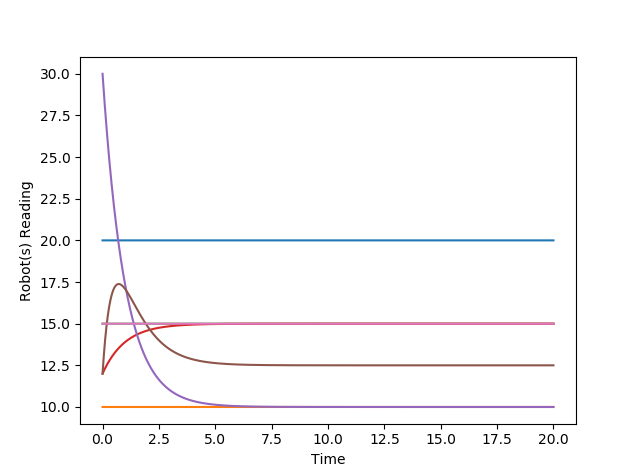
Switch time = 2s

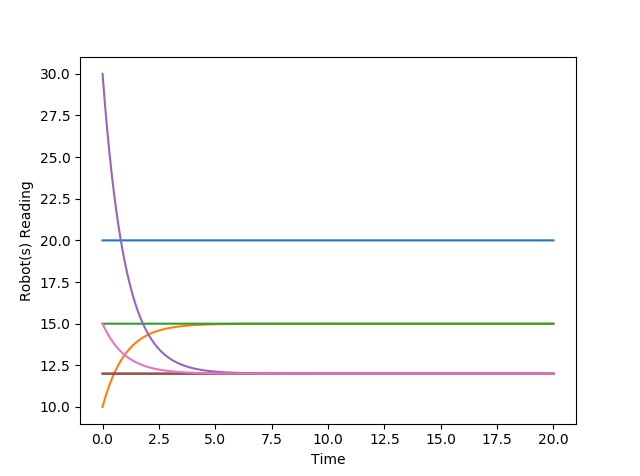
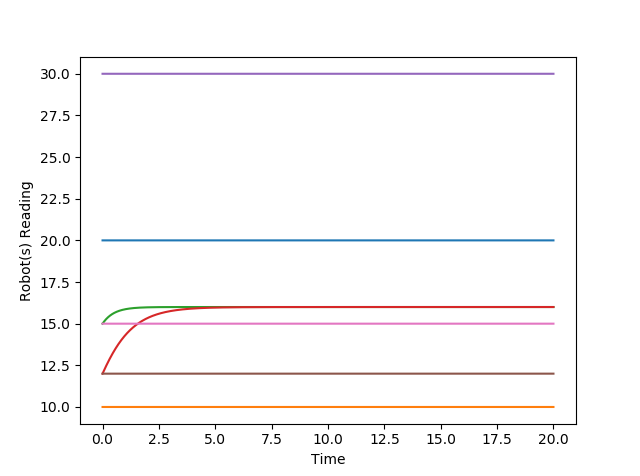
We can see that when switch time is less, then the curves are smoother.

1. **1) No**, for each graph the consensus protocol will **not** **converge** since, in all the cases the graph **doesn’t have rooted out branching**

**2) Yes**, with switching, the consensus protocol will **converge**  because at all the time the **union** of all the **graphs contain rooted out branching**.

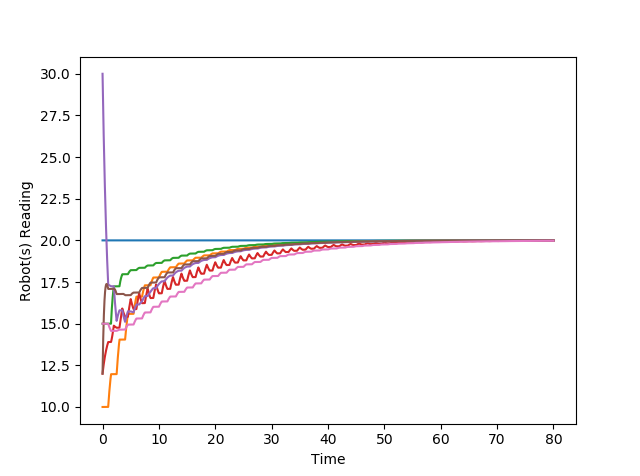
**3)** Whenno switching,

****

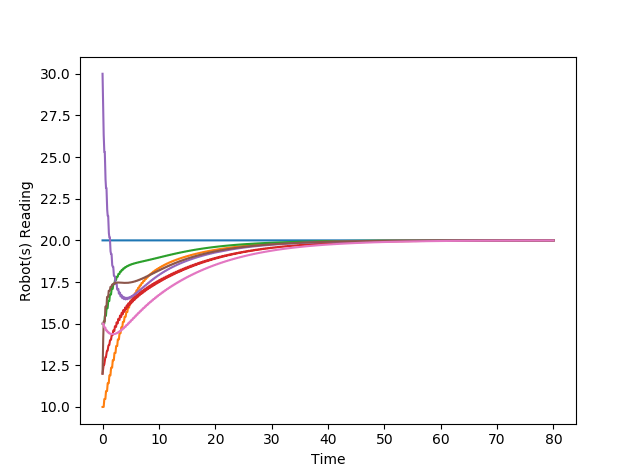
**** Graph(a)

Graph(b)

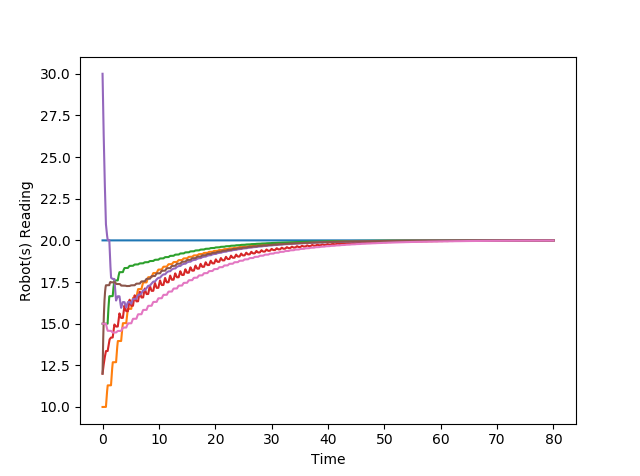
Graph(c)

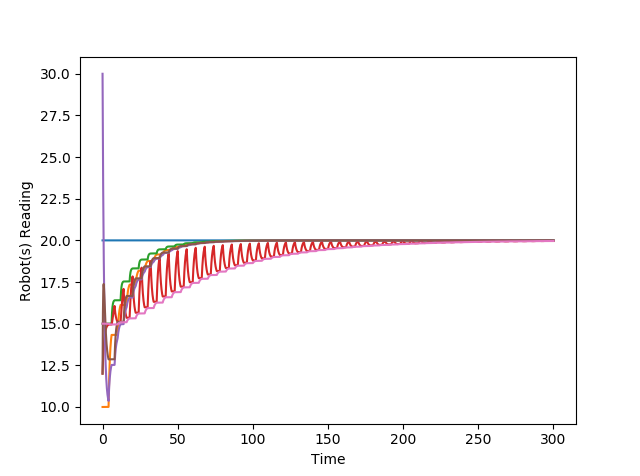
 When switching time = 0.5s

**4)** Following are the plots

 Switch time = 0.1 s

Switch time = 0.3 s





Switch time = 2s

We can see that which switching time less convergence is faster.